Pergamon

Int. J. Heat Mass Transfer. Vol. 39, No. 4, pp. 779–789, 1996

0017-9310/96 $15.00+0.00

0017-9310(95)00158-1

Experimental study of free convection and radiation in horizontal fin arrays

V. RAMMOHAN RAO and S. P. VENKATESHAN†

Department of Mechanical Engineering, Indian Institute of Technology, Madras 600 036, India

(*Received* 31 *January* 1994 *and in final form* 12 *April* 1995)

Abstract—An experimental investigation of interaction of free convection and radiation in a horizontal fin array is carried out. A differential interferometer is used to obtain free convection heat transfer and radiation is calculated by solving the integro-differential equations numerically. Results are presented to show the effects of various parameters such as emissivity of the fin surfaces, fin spacing, fin height and base temperature. Correlations are suggested in terms of non-dimensional parameters, based on a large number of experiments. The main conclusion to be drawn from the present study is that radiation–convection interaction invalidates additive approaches in which convection and radiation contributions are independently calculated assuming all surfaces to be isothermal and then adding these to obtain the total heat loss from the fin array.

1. INTRODUCTION

Early experimental work, in the 1960s [1–5], on free convection in horizontal and vertical fin arrays dealt with cases where surface radiation could be ignored. These concentrated on polished metal extended surfaces with low emissivities. With electronic component cooling providing an impetus to work in this area there was a resurgence of activity in this area, radiation also getting a share of attention. One of the earliest in this genre was the work of Edwards and Chaddock [6] who concluded that radiation could contribute as much as a third to the total heat transfer from cylindrical fins of surface emissivity of 0.99. Even for polished aluminum case radiation contribution was between 10 and 20% [7].

Calculations of Donovan and Rohrer [8] showed that finning could either increase or decrease the total heat loss when radiation was dominant. The interaction between the fin, the base, adjacent fins and the ambient strongly influences the thermal performance of the fin array. Van de Pol and Tierney [9] have presented an approximate technique, useful as a design tool, for the analysis of heat transfer by free convection and radiation in vertical fin arrays. Correlations applicable to free convection from a *U*-shaped channel, vertical flat plate and radiation from a *U*-shaped channel were made use of, assuming no interaction between convection and radiation. Rae and West [10] presented data for thermal radiation from finned heat sinks based on the apparent emissivity concept [11], assuming the fins to be isothermal. Comparison with experiments conducted on a commercial heat sink indicated a good agreement between the experimental data and the calculated values.

† Author to whom correspondence should be addressed.

Numerical results of Saikhedkar and Sukhatme [12, 13] included the effect of interaction between radiation and convection, a clear improvement over the approach used in [8]. They proposed correlations for Nusselt number in terms of Grashof number, fin length, fin spacing, emissivity and the temperature. While the convective heat transfer showed an increase, the radiative component showed a decrease with an increase in the Grashof number. Manzoor *et al.* [14] analyzed, via 1D and 2D approaches, heat transfer by convection and radiation for radiatively interacting black surfaces. Experiments on highly populated horizontal pin fins fixed to a vertical base plate [15] indicated that (a) fins enhance heat transfer by as much as a factor of six, as compared to an unfinned surface and (b) radiation could contribute between 25 and 45% of the total heat loss. In a later paper [16] the same authors have studied the effect of orientation of the fin array. Radiation was estimated based on isothermal fins, and convection by heat balance. Experiments of Gugliemini *et al.* [17] showed that heat transfer performance of staggered array of discrete vertical plates of two emissivities (0.05 and 0.85) is superior to that from U-shaped channels. Recently Aihara *et al.* [18] studied the performance of a pin fin array fixed to a vertical base plate. Flow visualization coupled with velocity profile measurements indicated a similarity with rectangular fin arrays. Radiation was estimated based on the apparent emissivity concept. A general formula for apparent emissivity was also presented.

The conclusion to be drawn from the previous literature is that the coupling between radiation and free convection has not received the attention it deserves. Assumption of isothermal fin surfaces eliminated such coupling. This may have some justification for short fins. However, it is well known [3, 9, 15] that

NOMENCLATURE

dF	view factor between an infinitesimal area and a finite area
d^2F	view factor between two infinitesimal areas
H	fin height [m]
G_f, G_b	irradiation on the fin and base, respectively [W m^{-2}]
k	thermal conductivity [W m^{-1}K^{-1}]
K	differential interferometer constant, instrument setting dependent, 0.139 or 0.0695 or 0.046 [m^{-1}K^{-1}]
m	fringe shift factor, the ratio of fringe deflection to fringe spacing
Nu	Nusselt number, hS/k_a
N_{rc}	radiation conduction parameter, $2\sigma\varepsilon_f T_b^3 H^2/k_f t$
Pr	Prandtl number, ν/α
Q	heat transfer rate [W]
Ra_s	Rayleigh number, $g\beta(T_b - T_\infty)S^3/\alpha\nu$
S	fin spacing [m]
T	temperature [K]
t	fin thickness [m]
W	width of the fin flats [m]
x	coordinate along the fin height [m]
y	coordinate normal to the fin flat [m].

Greek symbols

α	thermal diffusivity of air [m^2 s^{-1}]
β	coefficient of volumetric expansion of air [K^{-1}]
ε	hemispherical emissivity
ν	kinematic viscosity of air [m^2 s^{-1}]
σ	Stefan–Boltzmann constant [5.67×10^{-8}W m^{-2}K^4].

Subscripts

a	refers to air
b	refers to the base
c	convective
f	refers to the fin material
∞	refers to the ambient
r	radiative
t	total, i.e. convective + radiative.

tall fins exhibit better heat transfer characteristics than short fins. One can expect significant temperature variation in the case of tall fins and the isothermal assumption is not justified. Since such fin temperature variation is a consequence of an interplay between conduction in the fin material, free convection and radiation at the fin surface, there is bound to be an interaction between these three modes of heat transfer. This aspect is taken up for a detailed examination in the present study.

2. EXPERIMENTAL DETAILS

The experimental technique employed in the present study has been adequately dealt with by earlier publications from this laboratory [19–21]. The method basically consists in measuring the convective heat transfer using a differential interferometer and estimating the radiative heat transfer by the solution of an integro-differential equation. The fin temperature profile is obtained by using the measured base and tip temperatures and by numerically solving the fin equation. Because of the interaction between the three modes of heat transfer referred to earlier all the equations have to be solved simultaneously. The fin system used in the present study is shown in a cross sectional view in Fig. 1(a). An exploded pictorial view is shown in Fig. 1(b). The positions of the thermocouples as well as the main dimensions are included therein. A more detailed description of the apparatus is available elsewhere [22]. By the choice of suitable spacer blocks the fin spacing may be fixed at four different values, namely 10, 15, 20 and 25 mm. All the

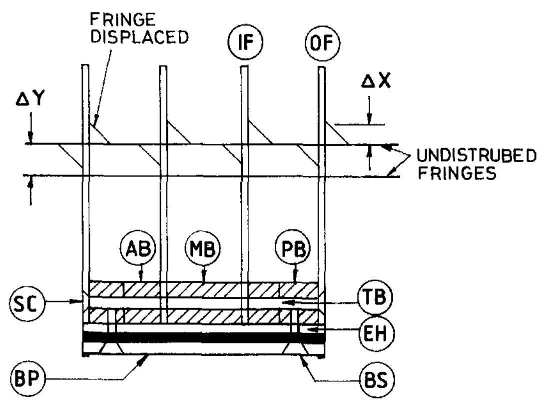

Fig. 1.(a) Details of the assembly of the array of four vertical rectangular fins attached to a horizontal base maintained at a uniform temperature. Key to the symbols are: AB, additional block; BP, base plate; BS, base screw; EH, electric heater; IF, inner fin; MB, middle block; OF, outer fin; PB, permanent block; SC, side screw; TB, tie bar and T, thermocouple.

fins are made from 1.5 mm aluminum plates 1.5 mm thick and three heights were employed, namely 30, 50 and 70 mm. The width of all the fins were fixed at 50 mm. Fin surfaces were prepared such that the emissivity could take on the values of 0.05, 0.18, 0.47 and 0.85 [22]. However, the spacer blocks had a constant emissivity of 0.85 in all the experiments reported here. The electrical input to the heater could be adjusted such that base temperature could be adjusted anywhere between 320 and 390 K. Table 1 summarizes the range of parameters used in the present study.

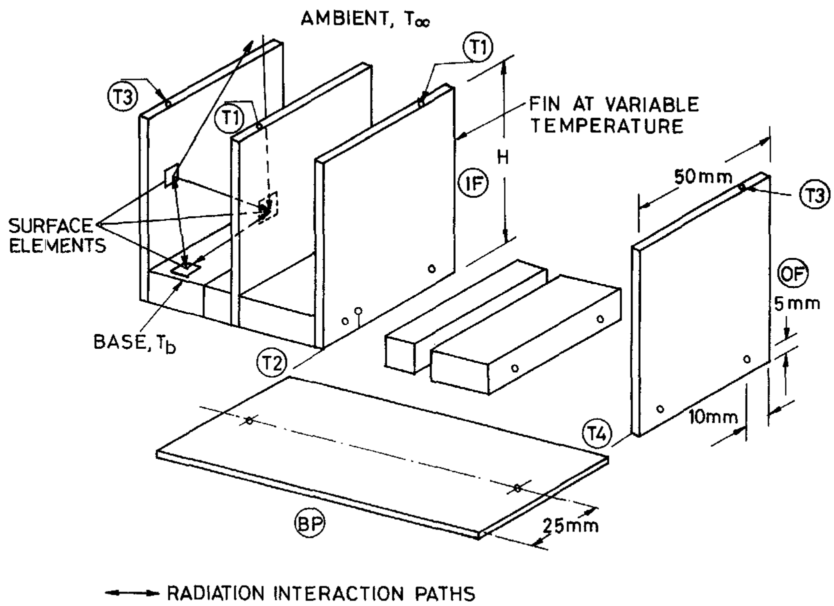

Fig. 1.(b) Pictorial view of the fin array assembly showing the radiative interaction paths as well as the salient dimensions.

Table 1. Range of parameters used in the experimental study

Parameter	Range	Remark
Fin height [mm]	30, 50, 70	
Fin spacing [mm]	10, 15, 20, 25	The total number of
Emissivity	0.05, 0.18, 0.47, 0.85	experiments is
Heating level	To get five base temperatures such that 320 K < T_b < 390 K	$3 \times 4 \times 4 \times 5 = 240$
Fin thickness [mm]	1.5	
Fin width [mm]	50	
Fin thermal conductivity [W $m^{-1}K^{-1}$]	205 (aluminum)	All these are held fixed
Emissivity of base surface	0.85 (black board paint)	
Number of fins in the array	4	

A brief note on the placement of the thermocouples are in order now. All the thermocouples were of 0.2 mm diameter copper constantan pair. At the tip these were located at a distance of 12 mm from the closest edge so that it indicated the mean temperature. The base temperature did not vary by more than ± 0.2 K while the end to end variation of the tip temperature was within ± 1.0 K. The temperature at about 12 mm from either end was equal to the average temperature in all the cases.

3. ANALYSIS OF DATA

The experiments were all performed in the steady state and the interferograms were photographed when the thermocouple outputs had stabilized. From the interferogram the fringe shifts at all the fin surfaces were measured. The governing equation for temperature along any one of the fins can be written down as under.

$$d^2T/dx^2 + \{Kk_a/k_f t\}(m_l + m_r)T^2 + \{q_{rl} + q_{rr}\}/k_f t = 0 \tag{1}$$

where m_l and m_r are the left and right hand side fringe shift factors for the fin under consideration, q_{rl} and q_{rr} are the left and right hand side radiative heat fluxes, respectively and K is the differential interferometer constant [21]. One dimensional temperature field is justified on the basis of temperature measurements which indicated a variation of less than ± 1.0 K in the crosswise direction. Isothermal assumption often made in the literature was rejected on the basis of measurements which showed significant temperature variation along the height. The radiative terms in equation (1) are based on an enclosure analysis [22]

Area elements and radiation interaction paths are shown schematically in Fig. 1(b). As an example, the irradiation on the right side of fin 2 may be written as

$$G_{f2r} = dF_{2-\infty}\sigma T_\infty^4 + \int_0^S d^2F_{2-b}[\varepsilon_b\sigma T_b^4 + (1-\varepsilon_b)Gb_1]\,dS + \int_0^H d^2F_{2-1}[\varepsilon_f\sigma T_1^4 + (1-\varepsilon_f)Gf_{1l}]\,dH. \quad (2)$$

With this the q_{rl} in equation (1) is given by

$$q_{rl} = \varepsilon_f[\sigma T_2^4 - G_{f2r}]. \quad (3)$$

Similarly appropriate relations are derived for q_{rr} also. The fin equation thus needs the solution of an integro-differential system of equations. Elemental shape factors appearing in the above (the dFs) are all obtained by the use of the contour integration method [23]. The non-linear nature of the above equations necessitates an iterative solution. A Gauss–Siedel point by point iterative procedure was used for solving both the fin and the integral equations. These equations were discretized by finite differences and solved explicitly. The fin temperatures were initialized by a linear variation with height based on the measured base and tip temperatures. With this initial profile the irradiations are all calculated. Using these irradiations and the measured fringe shift factors temperatures are updated by solving the fin equations. The procedure is then repeated till convergence.

Turning attention to the heat transfer from the base, we use the radiation fluxes estimated from the above along with convective heat transfer obtained by measuring the fringe shift factors in a companion experiment in which the fringes were oriented normal to the base. It was observed that the mean fringe shift (the base is at a uniform temperature) could be used to calculate the convective component. A dimensional analysis of the equations shows that the governing non-dimensional parameters are: Rayleigh number Ra_s, radiation conduction interaction parameter N_{rc}, the aspect ratios S/W and H/W and the surface emissivity ε_f.

As far as the heat transfer from the lateral edges of the fins are concerned, no effort has been made to measure the convective component. The top edge heat transfer is implicitly included because the fin temperature is calculated by solving a boundary value problem utilizing measured base and tip temperatures. However, the radiative component has been taken into account. In the sections to follow, radiative heat loss from the fin flats include the edge heat loss due to radiation. However the convective heat loss from the fin flats do not include the edge contributions. If one wants to include the edge contributions to convective heat loss also from a fin array of the present kind, these may be estimated using the expressions given by Aihara [24] and added to the the present total heat loss. Even though the orientation of the array in [24] is different from that in the present study the edges are similarly disposed with respect to the air flow.

4. RESULTS AND DISCUSSION

A few typical results for an inner fin in the array are presented to bring out the effect of the various parameters on its thermal performance. The experimental data is presented in a usable form as correlations. Later the thermal performance of outer fins are compared with those of the inner fins. Comparisons with previous studies and a general discussion on the nature of interaction between convection and radiation rounds off the discussion.

4.1. *Heat transfer performance of an inner fin in the array*

Figure 2 depicts the variation of the temperature, and the various heat fluxes (as labelled) along the fin flat for the case of the tallest fin ($H = 70$ mm) with the widest spacing ($S = 25$ mm), highest emissivity ($\varepsilon_f = 0.85$) and for a base temperature of 382 K. These variations are, in general, typical of variations observed in all the experiments performed in the present study. All the fluxes are relatively small near the base. Whereas the radiative flux shows a monotonic increase as we move away from the base, convective as well as the total heat fluxes show a non-monotonic behavior. Invariably there is observed a local maximum roughly at one third the height of the fin flat and a second one at the fin tip. Such a behavior was observed in the earlier experiments of Sobhan *et al.* [19, 20] also. The temperature drop of 14 K from the base to the tip is significant.

Comparison of results for two different fin heights shown in Fig. 3 indicates that short fins are subject to much smaller variation of temperature along the fin. Heat fluxes are strong functions of fin height with short fins showing much larger heat fluxes, both radiative and convective. Generally the convective fluxes are larger than the radiative fluxes. Because short fins run hotter and the shape factors to the ambient are more favorable, the radiative flux increases more sharply in the case of short fins.

Turning attention to the effect of fin spacing on heat transfer, it was observed that heat loss from the fins have a weak dependence on fin spacing. When the fin emissivity is increased, the radiative heat loss showed an increase with fin spacing. This is easily explained as due to an increase in the view factor between the fin and ambient when S is increased. The variation with spacing was, however, very weak in the case of small values of emissivity. In these cases there are two opposing mechanisms that are present, namely the increase in equivalent emissivity due to the cavity effect (due to increased role of inter reflections) for smaller spacings with a simultaneous decrease in the view factor between the fin and the ambient. These two factors are more or less evenly balanced and hence

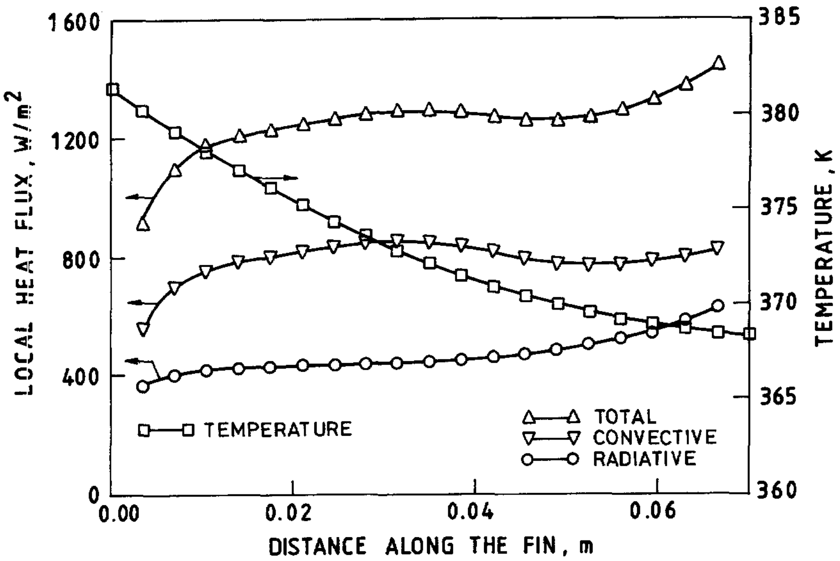

Fig. 2. Variations of convective, radiation, total heat fluxes and temperature along the fin for $H = 70$ mm, $S = 25$ mm, $\varepsilon_f = 0.85$ and $T_b = 382$ K.

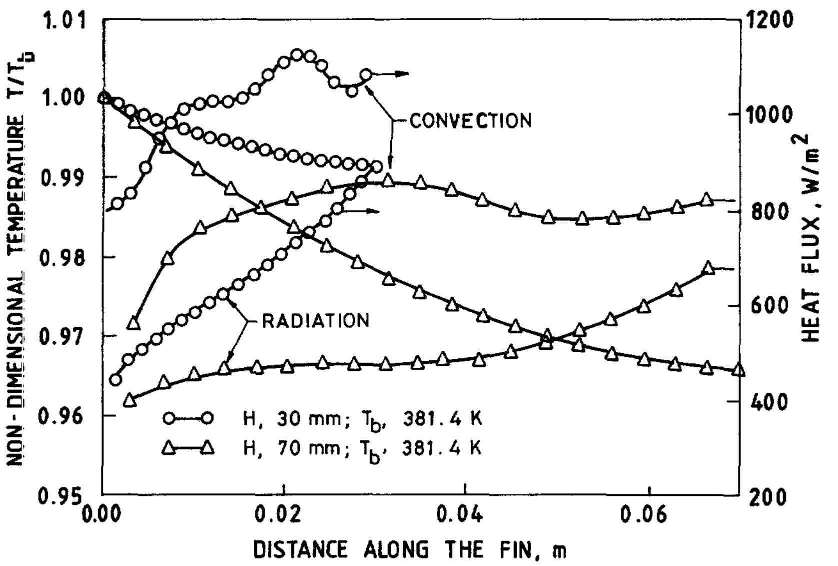

Fig. 3. Comparison of heat flux and temperature variations along the fin height for fins of two different heights with $S = 25$ mm, $\varepsilon_f = 0.85$ and $T_b = 381.4$ K.

the radiant heat loss remained more or less invariant with a change in the fin spacing.

Effect of base temperature on heat transfer from fin flats is shown in Fig. 4. The data for all the emissivities lie within a narrow band indicated in this figure. This behavior is due to the complex interaction between free convection and radiation. With an increase in emissivity the radiant flux shows an increase. The convection component, however, shows a reduction due to the interaction effect since the average temperature of the fin is lowered due to the interaction.

The variation of the heat transfer from the base, in general, is found to be monotonic with respect to the emissivity, fin spacing, fin height and the base temperature levels. Fin flat emissivity increase caused a marginal reduction in the convective heat transfer whereas a substantial decrease took place for the radiative part. The latter is easily explained as due to an increase in the irradiation on the base due to an increased emission from the fin flats. Increase in fin height, likewise, reduced both the convective as well as radiation heat transfers. Convective heat transfer showed a linear increase with fin spacing while the radiation heat transfer showed a somewhat more pronounced non-linear effect. The base temperature increase showed a monotonic increase in both modes

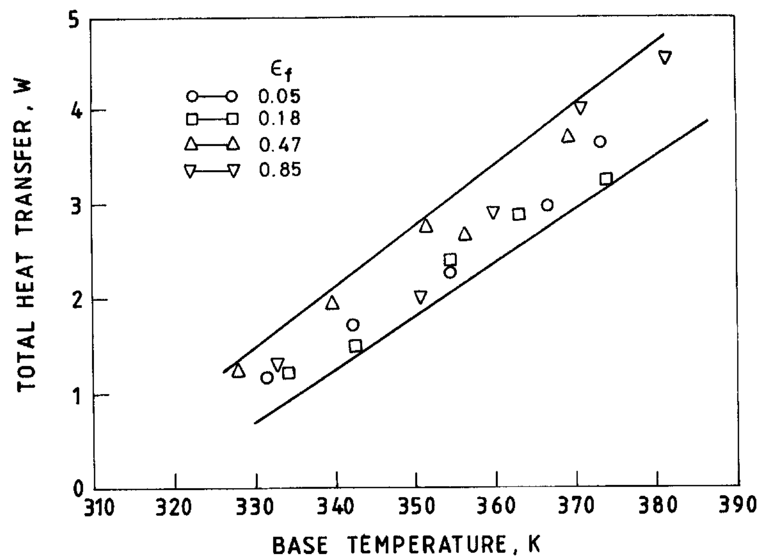

Fig. 4. Effect of base temperature on total heat loss from the fin flats for various emissivities and $S = 25$ mm, $H = 70$ mm.

of heat transfer. The reduction in the convective part with an increase in the fin flat emissivity was marginal.

4.2. *Heat transfer correlations*

Based on the large amount of data which was gathered through some 240 experiments it has been possible to arrive at useful correlations. Separate correlations are given for the fin flats and the base.

Fin flat heat transfer:

$$Nu_{\mathrm{ft}} = 0.791(1+N_{\mathrm{rc}})^{-2.483}Ra_{\mathrm{s}}^{0.214}(1+\varepsilon_{\mathrm{f}})^{0.572}(S/W)^{0.468} \quad (4)$$

with a correlation coefficient of 0.981 and a standard error of 0.034.

Base heat transfer:

$$Nu_{\mathrm{bt}} = 2.301(1+N_{\mathrm{rc}})^{-0.682}Ra_{\mathrm{s}}^{0.118}(1+\varepsilon_{\mathrm{f}})^{-0.47}(S/W)^{0.635}(H/W)^{-0.086} \quad (5)$$

with a correlation coefficient of 0.996 and a standard error of 0.017. The above two correlations are valid for the following range of parameters:

$$0.006 \leqslant N_{\mathrm{rc}} \leqslant 0.10 \quad 1500 \leqslant Ra_{\mathrm{s}} \leqslant 620000$$
$$0.05 \leqslant \varepsilon_{\mathrm{f}} \leqslant 0.87 \quad 0.2 \leqslant (\mathrm{S/W}) \leqslant 0.5$$
$$0.6 \leqslant (\mathrm{H/W}) \leqslant 1.4. \quad (6)$$

Parity plots shown in Fig. 5(a) and (b) indicate that 95% of the data lie within the $\pm 10\%$ error brackets shown thereon. A detailed error analysis presented in [22] indicates that the uncertainties in raw data are limited to a $\pm 5\%$ band. In these correlations radiation has been linearized with an attendant linearization error of less than 2%. Correlations have been provided separately for the convective and the radiative components in [22]. Those correlations indicate that fin spacing has a weak effect on convection whereas it has a strong effect on radiation, from the fin flats. Surface emissivity strongly affects radiant heat flux with an exponent of 0.603 which is close to a square root dependence [25] for the case of purely radiating fins of similar geometry. Base heat transfer shows a weak dependence on H, a strong dependence on S and a decrease with increasing ε_{f}.

In all the correlations given above and in [22], the exponent on the temperature difference is close to 0.25, a characteristic which is commonly encountered for laminar free convection heat transfer.

4.3. *Apparent emissivity; comparisons*

Aihara *et al.* [26] have expressed radiation heat loss from a fin array in terms of the apparent emissivity for a U-shaped channel consisting of isothermal surfaces of equal emissivities under the assumption of uniform irradiation–radiosity over the surfaces. In order to afford a comparison, their results were recalculated for the present geometry with base of the U being at a fixed emissivity of 0.85 while the emissivities of the limbs of the U could be varied. These results are referred to as $\varepsilon_{\mathrm{a1}}$ and have been calculated for all the cases for which experiments have been performed in the present study. Based on the present experimental results a second apparent emissivity designated as $\varepsilon_{\mathrm{a2}}$ has been calculated using the expression

$$\varepsilon_{\mathrm{a2}} = \frac{[Q_{\mathrm{r}}(\text{flats+edges})+Q_{\mathrm{r}}(\text{base})]}{\sigma(S+t)(2H+W)(T_{\mathrm{b}}^4-T_{\infty}^4)}. \quad (7)$$

A third apparent emissivity designated as $\varepsilon_{\mathrm{a3}}$ has been calculated, based on isothermal fin flats but with nonuniform irradiation–radiosity along fin height and the base. The difference between $\varepsilon_{\mathrm{a2}}$ and $\varepsilon_{\mathrm{a3}}$ will then be due only to the effect of nonuniform fin temperature while the difference between $\varepsilon_{\mathrm{a1}}$ and $\varepsilon_{\mathrm{a3}}$ will be due only to the uniform irradiation–radiosity assumption.

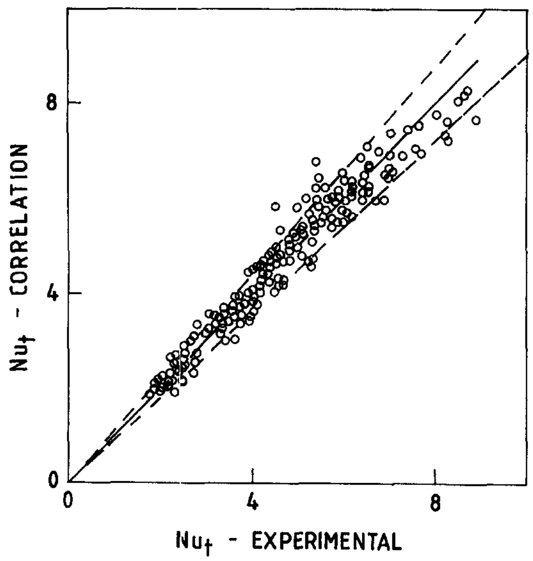

Fig. 5.(a) Parity plot showing the total Nusselt number obtained from the experiments against those calculated from the correlations for the fin flats.

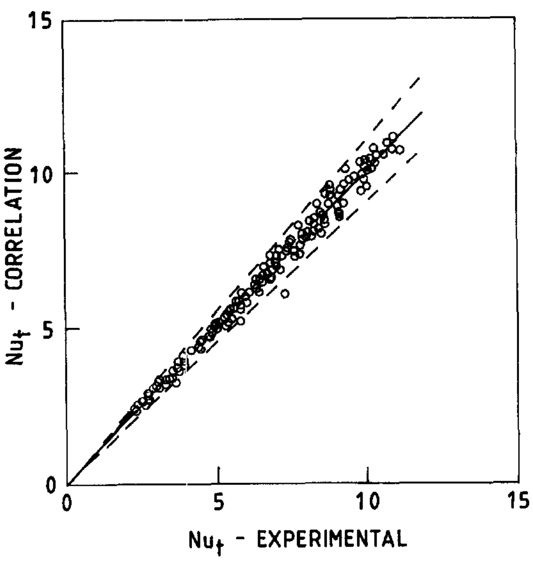

Fig. 5.(b) Parity plot showing the total Nusselt number obtained from the experiments against those calculated from the correlations for the base.

Table 2. Comparison of apparent emissivities calculated by three different methods explained in the text

ε_f	$\frac{S}{W}$	$\frac{H}{W}$	ε_{a2} Present	ε_{a1} Aihara	ε_{a3} Accurate
		0.6	0.359	0.372	0.365
	0.2	1.0	0.328	0.326	0.340
		1.4	0.308	0.302	0.331
		0.6	0.355	0.361	0.359
	0.3	1.0	0.303	0.303	0.313
0.05		1.4	0.275	0.271	0.291
		0.6	0.356	0.359	0.359
	0.4	1.0	0.294	0.292	0.301
		1.4	0.259	0.256	0.271
		0.6	0.359	0.360	0.361
	0.5	1.0	0.289	0.287	0.294
		1.4	0.250	0.247	0.260
		0.6	0.479	0.502	0.492
	0.2	1.0	0.469	0.493	0.498
		1.4	0.459	0.492	0.508
		0.6	0.448	0.460	0.457
	0.3	1.0	0.420	0.434	0.440
0.18		1.4	0.400	0.423	0.438
		0.6	0.433	0.439	0.439
	0.4	1.0	0.390	0.399	0.406
		1.4	0.365	0.382	0.394
		0.6	0.423	0.426	0.428
	0.5	1.0	0.372	0.377	0.384
		1.4	0.341	0.354	0.365
		0.6	0.647	0.678	0.672
	0.2	1.0	0.644	0.692	0.693
		1.4	0.637	0.703	0.710
		0.6	0.605	0.617	0.619
	0.3	1.0	0.587	0.62	0.626
0.47		1.4	0.567	0.627	0.637
		0.6	0.570	0.577	0.582
	0.4	1.0	0.550	0.569	0.577
		1.4	0.521	0.571	0.582
		0.6	0.546	0.549	0.555
	0.5	1.0	0.520	0.531	0.540
		1.4	0.483	0.528	0.539
		0.6	0.793	0.802	0.814
	0.2	1.0	0.797	0.816	0.829
		1.4	0.796	0.822	0.84
		0.6	0.745	0.748	0.761
	0.3	1.0	0.743	0.759	0.773
0.85		1.4	0.716	0.768	0.783
		0.6	0.703	0.705	0.719
	0.4	1.0	0.694	0.711	0.725
		1.4	0.652	0.719	0.734
		0.6	0.671	0.671	0.684
	0.5	1.0	0.652	0.672	0.685
		1.4	0.622	0.677	0.691

The result of these calculations are presented in Table 2. Since ε_{a2} has a mild dependence on $(T_b - T_\infty)$, the values in the table are the average of results for five temperatures and thus correspond to $T_b = 360$ K and $T_\infty = 305$ K. The table indicates that ε_{a1} is generally greater than ε_{a2} excepting in a few cases of low emissivities. These are the cases where uniform irradiation–radiosity assumption is violated. For the same cases ε_{a1} and ε_{a3} show significant differences thus supporting our contention that the uniform irradiation–radiosity assumption may not be a good one to make. In order to make these calculations useful to the designer they have been correlated as under:

$$\varepsilon_{a1} = 0.590\varepsilon_f^{0.309}(S/W)^{-0.229}(H/W)^{-0.116} \quad (8)$$

$$\varepsilon_{a2} = 0.628\varepsilon_f^{0.294}(S/W)^{-0.204} (H/W)^{-0.162}(T_b - T_\infty)^{-0.024} \quad (9)$$

$$\varepsilon_{a3} = 0.601\varepsilon_f^{0.304}(S/W)^{-0.219}(H/W)^{-0.079}. \quad (10)$$

Parity plots (not given here) showed that all these correlations are valid with a $\pm 10\%$ error margin. Also the discrepancies between ε_{a1} and ε_{a2} are within 12%. This will make a difference of only 5% in the total heat transfer. On this basis it may be concluded that any one of the three correlations given by equa-

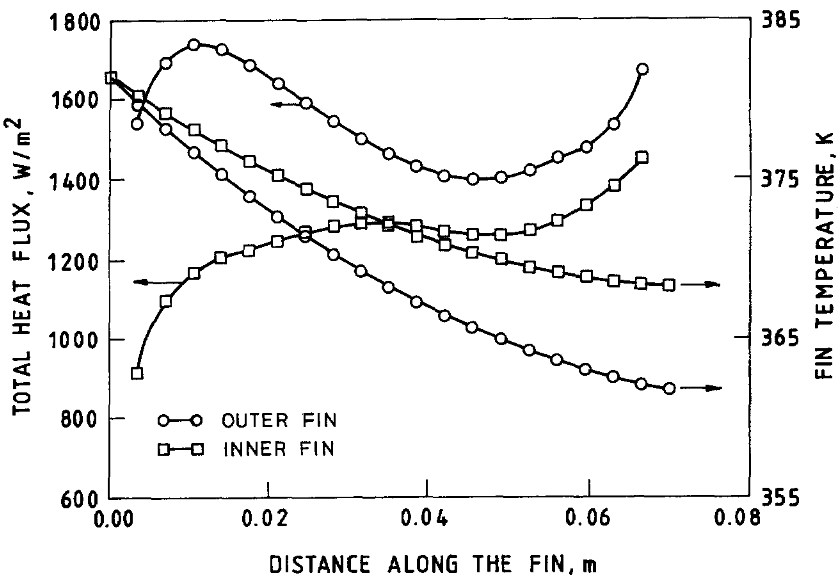

Fig. 6. Comparison of local heat fluxes and temperatures for an inner fin with an outer fin in the array for $H = 70$ mm, $S = 25$ mm and $\varepsilon_f = 0.85$.

tions (8)–(10) may be used for evaluating the apparent emissivity, at least in the range of parameters considered in the present study.

We have presented above the thermal performance data for the inner fins in a four fin array. It is expected that the inner fins of such an array approximate any inner fin in an array consisting of several fins. Calculations presented in [25] show that the temperature profiles for all the inner fins in an array have similar profiles and fall within a narrow band as opposed to the end fins which show a markedly different behavior. Hence it appears reasonable to use the correlations provided earlier for all the intermediate fins in an array.

The present study has also provided data on all the fins in the four fin array including the end fins. It is possible to compare the performance of the end fins with the performance of the intermediate fins. The basic difference between the inner and end fins is that the inner fin is subjected to a channel flow on both the sides while the end fins are subject to a channel fin on one side and free convective flow past a vertical non-isothermal surface. Secondly the end fins are exposed directly to the ambient on one side and thus do not have any radiative interaction with the base or a neighboring fin on one side. The temperature profiles shown in Fig. 6 indicate that the outer fins run cooler than the inner fins and also lose more heat than the inner fins. A detailed analysis of end fin data and a summary of these results in the form of correlations are given in [22]. Based on those correlations and the correlations for the inner fins and the base given earlier it is possible to estimate the total heat loss from a fin array.

As an example, a four fin array with $\varepsilon_f = 0.85$, $H = 0.07$ m, $S = 0.025$ m, $T_b = 381.4$ K and $T_\infty = 304$ K will dissipate 23.26 W. This total heat loss may be broken up into the following:

loss from two inner fins: 9.52 W (40.9%);

loss from the two intervening base areas: 2.52 W (10.8%) and

loss from two end fins: 11.22 W (48.3%).

Thus the total heat loss from the fin flats account for about 89% of the total heat loss from the array. Out of this radiation contributes 9.26 W or 40% to the total heat loss. An analysis of all the present experimental data shows that the percentage contribution of radiation to total heat loss from inner fins range from about 7 to 36% as shown in Table 3.

Table 3. Percentage contribution of radiation to the total heat loss from the array for the various values of the parameters used in the experiments

H	ε_f	$S = 0.01$	$S = 0.015$	$S = 0.02$	$S = 0.025$
0.03	0.05	7.3	8.1	8.7	9.2
	0.18	14.9	16.4	17.5	18.4
	0.47	24.8	26.9	28.5	29.8
	0.85	33.2	35.8	37.6	39.1
0.05	0.05	6.9	7.6	8.2	8.6
	0.18	14.0	15.4	16.4	17.3
	0.47	23.3	25.4	26.9	28.1
	0.85	31.4	33.8	35.5	36.9
0.07	0.05	6.5	7.2	7.8	8.2
	0.18	13.4	14.7	15.7	16.5
	0.47	22.3	24.3	25.7	26.9
	0.85	30.0	32.4	34.1	35.4

4.4. *Comparisons and general discussion*

Most of the earlier studies on heat transfer from fin arrays have assumed the fins to be isothermal thus

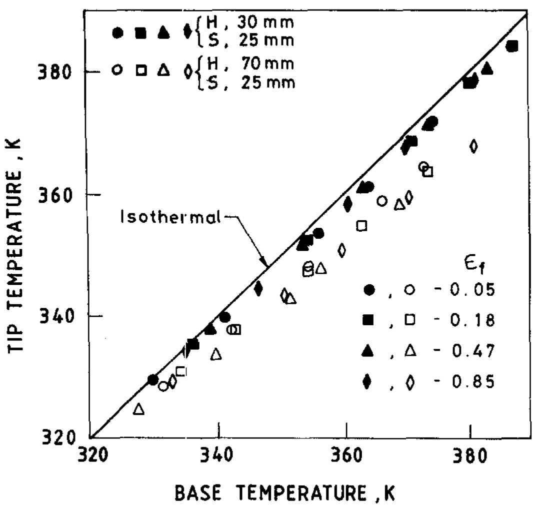

Fig. 7. Variation of tip temperature with the base temperature for various fin flat emissivities and for two different fin heights.

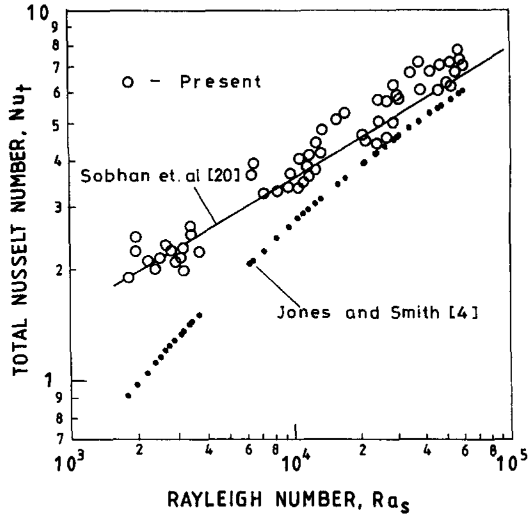

Fig. 8. Comparison of total Nusselt number results (fin flats and the base) with the predictions of correlations due to Sobhan *et al.* [20] and Jones and Smith [4].

avoiding any interaction that might exist between radiation and convection. This assumption will certainly overpredict radiation as has been seen earlier. This assumption should also overpredict free convection since it is known from previous studies [27, 28] that the interaction between radiation and convection tends to reduce the convective heat loss via the variation in the wall temperature. Significant deviations from isothermal conditions are observed in the experiments as shown in Fig. 7. The taller fins of 70 mm depart more than short fins from isothermal conditions, both for a fixed spacing of 25 mm. Results also indicate that inter fin spacing affects the temperature variation along fin height to a significant extent. In order to determine the error due to isothermal assumption in estimating the radiative heat transfer, a few calculations were made, by deliberately setting the tip temperature equal to the base temperature while analyzing the experimental data. The results showed that radiative component would be overestimated by as much as 15%. Assuming that the maximum contribution of radiation itself to total heat loss is some 40%, this translates to an overprediction of total heat loss by some 6%. From a practical engineering point of view this is not significant.

However the reduction in the convective heat loss due to interaction between convection and radiation can be as much as 30% when the emissivity changes from 0.05 to 0.85. In view of this, the isothermal assumption would affect the convective heat loss more severely than the radiative loss. This leads one to conclude that all those previous studies based on the isothermal assumption grossly overestimate the radiative as well as the convective part. These conclusions are further supported by the results shown in Fig. 9. When the emissivity of the fin flats is changed from 0.05 to 0.85 there is an increase in the total heat loss by a mere 10% which is much smaller than the radiation contribution which could be as high as 40%. The reduction in the convective component due to the interaction offsets a large part of the increase due to radiation.

Direct comparison of the present results with earlier literature is not possible since the present study is the first of its kind. However, in some special cases comparisons are indeed possible. Sobhan *et al.* [20] have presented data for polished metal fins (aluminum, brass and mild steel) for which $\varepsilon_f = 0.05$ and the radiation contribution was ignored on the plea that it was not more than about 5% of the total heat loss. Experiments were conducted for a horizontal fin array with fins of constant height of 70 mm. The results were correlated as

$$Nu_c\{k_a/k_f\}^{0.299} = 0.022\,Ra_s^{0.337} \tag{11}$$

with an error spread of $\pm 10\%$ and in the range $10^3 \leqslant Ra_s \leqslant 10^6$. Polished aluminum data from the present study are chosen for the sake of comparison. There are 60 data points available, and since the height was varied in the present study, there is a fair bit of scatter between the present data and the correlation equation (11), as shown in Fig. 8. It appears, from this comparison and our earlier discussion, that the reduction in convective heat loss due to interaction between radiation and convection is more or less compensated by the radiation heat loss itself.

Jones and Smith [4] have reported results based on experiments made on a horizontal fin array of polished aluminum wherein the fin height, spacing and the temperature levels were varied. Heat transfer measurements were made using a Mach–Zehnder inter-

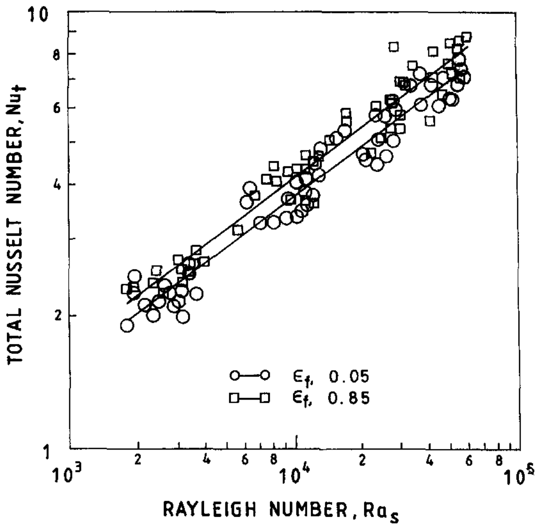

Fig. 9. Total Nusselt number variation with Rayleigh number for two extreme surface emissivities of the fins in the array.

ferometer. They correlated their data in the Elenbaas form as

$$Nu_c = 6.7\times10^{-4}[1-\exp\{-(7460/Ra_s)^{0.44}\}]^{1.7}. \quad (12)$$

In Fig. 8 this correlation also is shown. The present as well as Sobhan *et al.* correlation show close agreement with the results of Jones and Smith for large Rayleigh numbers. The discrepancy at lower Rayleigh numbers may be attributed to the neglect of heat capacity effects in [4] while evaluating heat loss from essentially unsteady measurements.

Donovan and Rohrer [8] considered the problem of radiation and convection in a conducting fin array (fins are not assumed to be isothermal) based on a specified constant convective heat transfer coefficient. From a parametric study, they concluded that the effectiveness of the fin decreases as the radiation component increases. This means that, even with an assumed constant heat transfer coefficient, the changes brought about by radiation interaction among the surfaces affect the temperature significantly and thus affect the convective heat loss. This is in broad conformity with the present findings.

Rae and West [10] advanced the concept of apparent emissivity based on a simple enclosure analysis and verified these by conducting experiments on vertical fin arrays in a vacuum. They did not include any effect of convection. The results of their study indicate that radiation heat transfer scales non-linearly with ε_f and show variation with respect to (T_b-T_∞) which is similar to the variation shown by the data in the present study.

Saikhedkar and Sukhatme [12] reported results of their numerical study of the conjugate problem of radiation and free convection in a vertical fin array consisting of rectangular fins oriented vertically on a vertical base. In this study they relaxed the isothermal fin assumption and the radiative transfer was based on a 2D model. It was shown that radiative heat transfer decreases with increase in Grashof number due to reduction in the fin temperature. It was concluded that the efficiency of the fins decreases with emissivity due to the same reason. These observations are in agreement with the present findings even though the geometry is different. The qualitative trends of radiative heat flux variation with respect to emissivity are in good agreement with those obtained in the present experimental study.

Sparrow and Vemuri [15, 16] considered natural convection and radiation heat transfer from highly populated pin fin arrays. They studied the effect of orientation and the number of pin fins in the population on heat transfer. Total heat transfer was measured by a calorimetric technique and the radiative part was calculated by the use of enclosure theory based on isothermal fins. The radiation part accounted for some 25–40% of the total heat loss. This apportioning is in broad agreement with the present measurements. However, the assumption of isothermal pin fins is a weakness of this study. It is possible that for the longer fins ($H = 75$ mm) the temperature variation may have been significant and hence the radiation component calculated by them may be substantially in error. However, the total heat transfer measured by calorimetry is dependable. A similar study using a similar technique was presented by Aihara *et al.* [18]. They present interesting flow visualization pictures that indicate a complex 3D flow pattern. All their experiments were done with a pin fin surface emissivity of 0.9.

5. CONCLUSIONS

This paper has presented results based on an interferometric study of heat transfer by free convection and radiation from a horizontal fin array, for a wide range of parameters. Results have been presented in the form of correlations useful for thermal design. Radiation results have been presented separately in terms of correlations for apparent emissivity. The most important conclusion to be drawn from the present study is that there is a mutual interaction between free convection and radiation and hence a simplistic approach based on additivity of radiation and convection heat transfer, calculated independently based on isothermal surfaces, is unsatisfactory. This conclusion is supported by several comparisons with work already available in the pertinent literature.

REFERENCES

1. K. E. Starner and H. N. McManus, An experimental investigation of free convection heat transfer from rectangular fin arrays, *ASME J. Heat Transfer* **85,** 273–278 (1963).
2. J. R. Welling and C. V. Wooldridge, Free convection

heat transfer co-efficients from rectangular fin arrays, *ASME J. Heat Transfer* **87,** 439–444 (1965).
3. F. Harahap and H. N. Mcmanus, Natural convection heat transfer from horizontal rectangular fin arrays, *ASME J. Heat Transfer* **89,** 32–38 (1967).
4. C. D. Jones and L. D. Smith, Optimum arrangement of rectangular fins on horizontal surfaces for free convection heat transfer, *ASME J. Heat Transfer* **92,** 6–10 (1970).
5. T. Aihara, Natural convection heat transfer from vertical rectangular fin arrays—III, *Bull. JSME* **13,** 1192–1200 (1970).
6. J. A. Edwards and J. B. Chaddock, An experimental investigation of the radiation and free convection heat transfer in cylindrical disk extended surfaces, *ASHRAE Trans.* **69,** 313–319 (1963).
7. J. B. Chaddock, Free convection heat transfer from vertical fin arrays, *ASHRAE J.* **12,** 53–60 (1970).
8. R. C. Donovan and W. M. Rohrer, Radiative and convective conducting fins on a plane wall, including mutual irradiation, *ASME J. Heat Transfer* **93,** 41–46 (1971).
9. D. W. Van de Pol and J. K. Tierney, Free convection heat transfer from vertical fin arrays, *IEEE Trans.* **PHP-10**(4), 267–271 (1974).
10. N. Rea and S. E. West, Thermal radiation from finned heat sinks, *IEEE Trans.* **PHP-12**(2), 115–117 (1976).
11. R. Siegel and J. R. Howell, *Thermal Radiation Heat Transfer*. McGraw-Hill, New York (1972).
12. N. H. Saikhedkar and S. P. Sukhatme, Heat transfer from rectangular cross-sectioned vertical fin arrays, *Proceedings of the Sixth National Heat and Mass Transfer Conference*, HMT-9-81, (1981).
13. S. P. Sukhatme, Heat transfer from rectangular vertical fin arrays. In *Current Research in Heat and Mass Transfer* (a compendium and a festschrift for Prof. Arcot Ramachandran), *Indian Soc. Heat Mass Transfer*, pp. 245–258 (1985).
14. M. Manzoor, D. B. Ingham and P. J. Heggs, The one-dimensional analysis of fin assembly heat transfer, *ASME J. Heat Transfer* **105,** 645–651 (1983).
15. E. M. Sparrow and S. B. Vemuri, Natural convection–radiation heat transfer from highly populated pin–fin arrays, *ASME J. Heat Transfer* **107,** 190–197 (1985).
16. E. M. Sparrow and S. B. Vemuri, Orientation effects on natural convection–radiation pin–fin arrays, *Int. J. Heat Mass Transfer* **29,** 359–368 (1986).
17. G. Guglielmini, E. Nannel and G. Tanda, Natural convection and radiation heat transfer from staggered vertical fins, *Int. J. Heat Mass Transfer* **30,** 1941–1948 (1987).
18. T. Aihara, S. Maruyama and S. Kobayakawa, Free convective–radiative heat transfer from pin–fin arrays with a vertical base plate (general representation of heat transfer performance), *Int. J. Heat Mass Transfer* **33,** 1223–1232 (1990).
19. C. B. Sobhan, S. P. Venkateshan and K. N. Seetharamu, Experimental analysis of unsteady free convection heat transfer from horizontal fin arrays, *Warme-und Stoffubertragung* **24,** 155–160 (1989).
20. C. B. Sobhan, S. P. Venkateshan and K. N. Seetharamu, Experimental studies on steady free convection heat transfer from fins and fin arrays, *Warme-und Stoffubertragung* **25,** 345–352 (1990).
21. V. Rammohan Rao, C. B. Sobhan and S. P. Venkateshan, Differential interferometry in heat transfer, *Sadhana* **15**(2), 105–128 (1990).
22. V. Rammohan Rao, Interferometric study of interaction between radiation and free convection in fins and fin arrays, Ph.D. Thesis, Indian Institute of Technology, Madras, India (1992).
23. E. M. Sparrow and R. D. Cess, *Radiation Heat Transfer*. Brooks and Cole, (1981).
24. T. Aihara, Natural convection heat transfer from vertical rectangular fin arrays—II, *Bull. JSME* **13,** 1182–1191 (1970).
25. S. Sunil Kumar, Venkatesh Nayak and S. P. Venkateshan, Optimum finned space radiators, *Int. J. Heat Fluid Flow* **14,** 191–200 (1993).
26. T. Aihara and S. Maruyama, Optimum design of natural cooling heat sinks with vertical rectangular fin arrays. In *Cooling Technology for Electronic Equipment* (Edited by W. Aung), pp. 35–54. Hemisphere, New York (1988).
27. Y. Asako and H. Nakamura, Heat transfer in a parallelogram shaped enclosure, *Bull. JSME* **25,** 1419–1427 (1982).
28. C. Balaji and S. P. Venkateshan, The effect of surface radiation on free convection in a square cavity, *Int. J. Heat Fluid Flow* **14,** 260–267 (1993).